02平面向量练习册

1. 平面向量数量积运算

1 已知$\overrightarrow{AB}=(2,3)$，$\overrightarrow{AC}=(3,t)$，$|\overrightarrow{BC}|=1$，则$\overrightarrow{AB}\cdot\overrightarrow{BC}=$（　）．

A. -3　　B. -2　　C. 2　　D. 3

答案 C

解析 $\because\overrightarrow{AB}=(2,3)$，$\overrightarrow{AC}=(3,t)$，

$\therefore\overrightarrow{BC}=\overrightarrow{AC}-\overrightarrow{AB}=(1,t-3)$，

$\because|\overrightarrow{BC}|=1$，

$\therefore\sqrt{1+(t-3)^2}=1$，

$\therefore t=3$，

$\therefore\overrightarrow{BC}=(1,0)$，

$\therefore\overrightarrow{AB}\cdot\overrightarrow{BC}$

$=2\cdot1+3\cdot0$

$=2$．

故选C．

2 已知单位向量$\vec{a}$，$\vec{b}$的夹角为60°，$\vec{a}-k\vec{b}$与$\vec{a}$垂直，则实数$k=$（　）．

A. 1　　B. -1　　C. 2　　D. -2

答案 C

解析 $\because$单位向量$\vec{a}$，$\vec{b}$的夹角为60°，

$\therefore\vec{a}\cdot\vec{b}=1\times1\times\cos60^\circ=\dfrac{1}{2}$，

$\because\vec{a}-k\vec{b}$与$\vec{a}$垂直，

$\therefore\left(\vec{a}-k\vec{b}\right)\cdot\vec{a}=\vec{a}^2-k\vec{a}\cdot\vec{b}=1-\dfrac{k}{2}=0$，

则实数$k=2$．

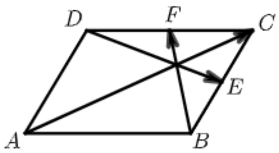
故选C．

3 已知平行四边形$ABCD$，$|\overrightarrow{AB}|=2$，$|\overrightarrow{AD}|=1$，E，F分别是线段BC，CD的中点，若$\overrightarrow{DE}\cdot\overrightarrow{BF}=-\dfrac{5}{4}$，则向量$\overrightarrow{AB}$与$\overrightarrow{AD}$的夹角为（　）．

A. $\dfrac{\pi}{6}$　　B. $\dfrac{\pi}{3}$　　C. $\dfrac{2\pi}{3}$　　D. $\dfrac{5\pi}{6}$

答案 B

解析 如图所示，



$\overrightarrow{DE}\cdot\overrightarrow{DF}=\left(\dfrac{1}{2}\overrightarrow{CB}-\overrightarrow{CD}\right)\cdot\left(\dfrac{1}{2}\overrightarrow{CD}-\overrightarrow{CB}\right)$

$=\dfrac{5}{4}\overrightarrow{CB}\cdot\overrightarrow{CD}-\dfrac{1}{2}\overrightarrow{CB}^2-\dfrac{1}{2}\overrightarrow{CD}^2=-\dfrac{5}{4}$，

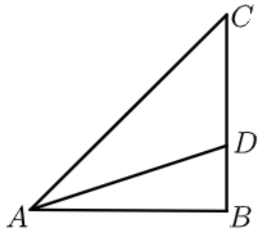
由$|\overrightarrow{CD}|=|\overrightarrow{AB}|=2$，$|\overrightarrow{BC}|=|\overrightarrow{AD}|=1$，得$\overrightarrow{CB}\cdot\overrightarrow{CD}=1$，

$\therefore\cos<\overrightarrow{CB}\cdot\overrightarrow{CD}>=\dfrac{1}{2}$，

$\therefore<\overrightarrow{CB}\cdot\overrightarrow{CD}>=\dfrac{\pi}{3}$．

故选B．

4 如图，在三角形ABC中，已知$AB=2$，$AC=3$，$\angle BAC=\theta$，点D为BC的三等分点．则$\overrightarrow{AD}\cdot\overrightarrow{BC}$的取值范围为（　）．



A. $\left(-\dfrac{11}{3},\dfrac{13}{3}\right)$　　B. $\left(\dfrac{1}{3},\dfrac{7}{3}\right)$　　C. $\left(-\dfrac{5}{3},\dfrac{55}{3}\right)$　　D. $\left(-\dfrac{5}{3},\dfrac{7}{3}\right)$

答案　D

解析　$\because \overrightarrow{AD}=\overrightarrow{AB}+\overrightarrow{BD}=\overrightarrow{AB}+\frac{1}{3}\overrightarrow{BC}=\overrightarrow{AB}+\frac{1}{3}\left(\overrightarrow{AC}-\overrightarrow{AB}\right)$

$=\frac{2}{3}\overrightarrow{AB}+\frac{1}{3}\overrightarrow{AC}$，

$\therefore \overrightarrow{AD}\cdot\overrightarrow{BC}=\left(\frac{2}{3}\overrightarrow{AB}+\frac{1}{3}\overrightarrow{AC}\right)=-\frac{2}{3}\left|\overrightarrow{AB}\right|^2+\frac{1}{3}\left|\overrightarrow{AC}\right|^2+\frac{1}{3}\overrightarrow{AB}\cdot\overrightarrow{AC}$

$=-\frac{2}{3}\times 4+\frac{1}{3}\times 9+\frac{1}{3}\times 2\times 3\cos\theta$

$=2\cos\theta+\frac{1}{3}$.

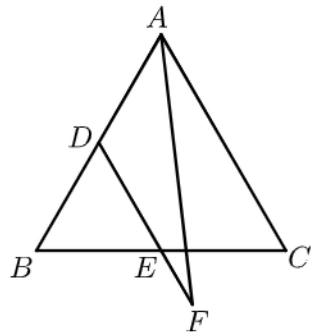
$\because -1<\cos\theta<1$，

$\therefore -\frac{5}{3}<2\cos\theta+\frac{1}{3}<\frac{7}{3}$.

$\therefore \overrightarrow{AD}\cdot\overrightarrow{BC}\in\left(-\frac{5}{3},\frac{7}{3}\right)$.

故选D .

5　如图，已知$\triangle ABC$是边长为1的等边三角形，点D，E分别是边AB，BC的中点，连接DE并延长到点F，使得$DE=2EF$，则$\overrightarrow{AF}\cdot\overrightarrow{BC}$的值为（　）．



A. $-\frac{5}{8}$　　B. $\frac{1}{8}$　　C. $\frac{1}{4}$　　D. $\frac{11}{8}$

答案　B

解析　方法一：$\overrightarrow{AF}\cdot\overrightarrow{BC}$

$=(\overrightarrow{AD}+\overrightarrow{DF})\cdot\overrightarrow{BC}$

$=\left(\frac{1}{2}\overrightarrow{AB}+\frac{3}{2}\overrightarrow{DE}\right)\cdot\overrightarrow{BC}$

$=\left(\frac{1}{2}\overrightarrow{AB}+\frac{3}{2}\overrightarrow{DB}+\frac{3}{2}\overrightarrow{BE}\right)\cdot\overrightarrow{BC}$

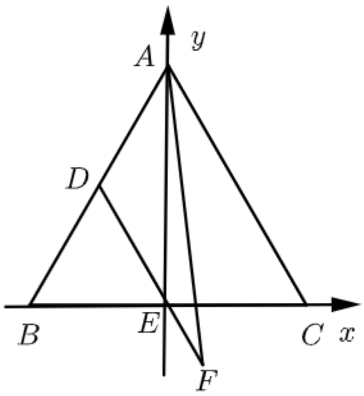
$=\left(\frac{5}{4}\overrightarrow{AB}+\frac{3}{4}\overrightarrow{BC}\right)\cdot\overrightarrow{BC}$

$=\frac{5}{4}\times 1\times 1\times\cos 120^\circ+\frac{3}{4}\times 1^2$

$=\dfrac{1}{8}$.

故选B .

方法二：建立如图所示的平面直角坐标系 .



由图知$B\left(\dfrac{1}{2},0\right)$, $C\left(\dfrac{1}{2},0\right)$, $A\left(0,\dfrac{\sqrt{3}}{2}\right)$,

所以$\overrightarrow{BC}=(1,0)$.

由题易知$DE=\dfrac{1}{2}AC$, $\angle FEC=\angle ACE=60^\circ$,

则$EF=\dfrac{1}{4}AC=\dfrac{1}{4}$,

所以点F的坐标为$\left(\dfrac{1}{8},-\dfrac{\sqrt{3}}{8}\right)$,

所以$\overrightarrow{AF}=\left(\dfrac{1}{8},-\dfrac{5\sqrt{3}}{8}\right)$.

所以$\overrightarrow{AF}\cdot\overrightarrow{BC}=\left(\dfrac{1}{8},-\dfrac{5\sqrt{3}}{8}\right)\cdot(1,0)=\dfrac{1}{8}$.

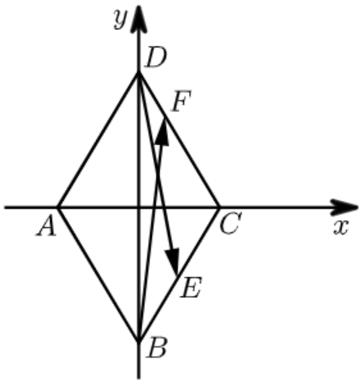
故选B .

6 已知菱形$ABCD$的边长为2 , $\angle BAD=120^\circ$, 点E , F分别在边BC , DC上 , $\overrightarrow{BE}=\dfrac{1}{2}\overrightarrow{BC}$, $\overrightarrow{DF}=\dfrac{1}{3}\overrightarrow{DC}$, 则$\overrightarrow{BF}\cdot\overrightarrow{DE}=$ (　) .

A. $-\dfrac{22}{3}$　　B. $\dfrac{2}{3}$　　C. $\dfrac{5}{6}$　　D. $\dfrac{22}{3}$

答案 A

解析 根据题意 , 分别以直线AC , BD为x , y轴 , 建立如图所示平面直角坐标系 ,



根据菱形$ABCD$的边长为2，$\angle BAD=120^\circ$可求出以下几点的坐标：$B\left(0,-\sqrt{3}\right)$，$C\left(1,0\right)$，

$D\left(0,\sqrt{3}\right)$，

$\because \overrightarrow{BE}=\frac{1}{2}\overrightarrow{BC}$，$\overrightarrow{DF}=\frac{1}{3}\overrightarrow{DC}$，

$\therefore \frac{|DF|}{|FC|}=\frac{1}{2}$，

$\therefore E\left(\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$，$F\left(\frac{1}{3},\frac{2\sqrt{3}}{3}\right)$，

$\therefore \overrightarrow{BF}=\left(\frac{1}{3},\frac{5\sqrt{3}}{3}\right)$，$\overrightarrow{DE}=\left(\frac{1}{2},-\frac{3\sqrt{3}}{2}\right)$，

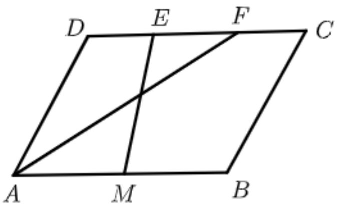
$\therefore \overrightarrow{BF}\cdot\overrightarrow{DE}=\frac{1}{6}-\frac{15}{2}=-\frac{22}{3}$．

故选：A．

7 在平行四边形$ABCD$中，$AD=1$，$\angle BAD=\frac{\pi}{3}$，点E，F在CD上，且满足$\overrightarrow{DE}=\frac{1}{3}\overrightarrow{DC}$，$\overrightarrow{DF}=\frac{2}{3}\overrightarrow{DC}$，若$M$为$AB$的中点，且$\overrightarrow{AF}\cdot\overrightarrow{ME}=1$，则$AB$的长为______．

答案 $\frac{9}{4}$

解析 如图，



$\overrightarrow{DE}=\frac{1}{3}\overrightarrow{DC}$，$\overrightarrow{DF}=\frac{2}{3}\overrightarrow{DC}$，且$\overrightarrow{DC}=\overrightarrow{AB}$，$\overrightarrow{MA}=-\frac{1}{2}\overrightarrow{AB}$，

$\therefore \overrightarrow{AF}=\overrightarrow{AD}+\overrightarrow{DF}$

$=\overrightarrow{AD}+\frac{2}{3}\overrightarrow{AB}$，$\overrightarrow{ME}=\overrightarrow{MA}+\overrightarrow{AD}+\overrightarrow{DE}$

$=-\frac{1}{2}\overrightarrow{AB}+\overrightarrow{AD}+\frac{1}{3}\overrightarrow{AB}$

$=-\frac{1}{6}\overrightarrow{AB}+\overrightarrow{AD}$，

且$\overrightarrow{AD}=1$，$\angle BAD=\frac{\pi}{3}$，$\overrightarrow{AF}\cdot\overrightarrow{ME}=1$，

$\therefore\left(\overrightarrow{AD}+\frac{2}{3}\overrightarrow{AB}\right)\cdot\left(-\frac{1}{6}\overrightarrow{AB}+\overrightarrow{AD}\right)$

$=\overrightarrow{AD}^2-\frac{1}{9}\overrightarrow{AB}+\frac{1}{2}\overrightarrow{AB}\cdot\overrightarrow{AD}$

$=1-\frac{1}{9}|\overrightarrow{AB}|^2+\frac{1}{4}|\overrightarrow{AB}|=1$，解得$|\overrightarrow{AB}|=\frac{9}{4}$或0（舍去），

$\therefore AB=\frac{9}{4}$．

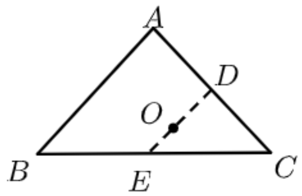
故答案为：$\frac{9}{4}$．

2. 综合问题

8 设点O在$\triangle ABC$的内部，且有$\overrightarrow{OA}+2\overrightarrow{OB}+3\overrightarrow{OC}=\vec{0}$，则$\triangle ABC$的面积与$\triangle AOC$的面积比为______．

答案 3

解析 分别取AC、BC的中点D、E，



$\because\overrightarrow{OA}+2\overrightarrow{OB}+3\overrightarrow{OC}=\vec{0}$，

$\therefore\overrightarrow{OA}+\overrightarrow{OC}=-2(\overrightarrow{OB}+\overrightarrow{OC})$即$2\overrightarrow{OD}=-4\overrightarrow{OE}$，

$\therefore O$是DE的一个三等分点，

$\therefore\frac{S_{\triangle ABC}}{S_{\triangle AOC}}=3$．

9 设M是$\triangle ABC$边BC上任意一点，N为AM的中点，若$\overrightarrow{AN}=\lambda\overrightarrow{AB}+\mu\overrightarrow{AC}$，则$\lambda+\mu$的值为（　　）．

A. 1　　B. $\frac{1}{2}$　　C. $\frac{1}{3}$　　D. $\frac{1}{4}$

答案 B

解析 设$\overrightarrow{BM}=t\overrightarrow{BC}$，

则有$\overrightarrow{AN}=\frac{1}{2}\overrightarrow{AM}=\frac{1}{2}\left(\overrightarrow{AB}+\overrightarrow{BM}\right)$

$=\frac{1}{2}\overrightarrow{AB}+\frac{1}{2}t\overrightarrow{BC}=\frac{1}{2}\overrightarrow{AB}+\frac{t}{2}\left(\overrightarrow{AC}-\overrightarrow{AB}\right)$

$=\frac{1-t}{2}\overrightarrow{AB}+\frac{t}{2}\overrightarrow{AC}$，

又$\overrightarrow{AN}=\lambda\overrightarrow{AB}+\mu\overrightarrow{AC}$，

所以$\begin{cases}\lambda=\frac{1-t}{2}\\ \mu=\frac{t}{2}\end{cases}$，有$\lambda+\mu=\frac{1-t}{2}+\frac{t}{2}=\frac{1}{2}$．

故选：B．

10 已知边长为2的菱形$ABCD$中，点F为BD上一动点，点E满足$\overrightarrow{BE}=2\overrightarrow{EC}$，$\overrightarrow{AE}\cdot\overrightarrow{BD}=-\frac{2}{3}$，则$\overrightarrow{AF}\cdot\overrightarrow{EF}$的最小值为（　　）．

A. $-\frac{2}{3}$　　B. $-\frac{4}{3}$　　C. $-\frac{152}{75}$　　D. $-\frac{73}{36}$

答案 D

解析 方法一：$\overrightarrow{BE}=\frac{2}{3}\overrightarrow{BC}$，设$\angle DAB=\theta$，

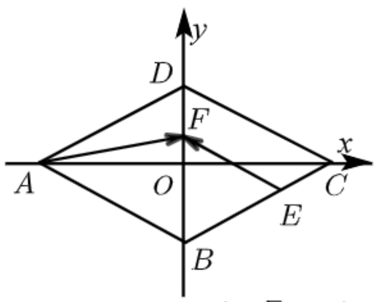
$\therefore\overrightarrow{AE}\cdot\overrightarrow{BD}=\left(\overrightarrow{AB}+\overrightarrow{BE}\right)\cdot\left(\overrightarrow{AD}-\overrightarrow{AB}\right)$

$=\overrightarrow{AB}\cdot\overrightarrow{AD}-\overrightarrow{AB}^2+\frac{2}{3}\overrightarrow{BC}\cdot\overrightarrow{AD}-\frac{2}{3}\overrightarrow{BC}\cdot\overrightarrow{AB}$

$=4\cos\theta-4+\frac{8}{3}-\frac{8}{3}\cos\theta=-\frac{2}{3}$，

$\therefore\cos\theta=\frac{1}{2}\Rightarrow\theta=\frac{\pi}{3}$，

以AC与BD交点为原点，AC为x轴，BD为y轴建立如下图所示的平面直角坐标系：



$\therefore A\left(-\sqrt{3},0\right)$，$E\left(\frac{2\sqrt{3}}{3},-\frac{1}{3}\right)$，设$F\left(0,t\right)$，

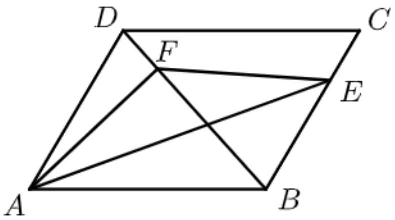
则$\overrightarrow{AF}=\left(\sqrt{3},t\right)$，$\overrightarrow{EF}=\left(-\frac{2\sqrt{3}}{3},t+\frac{1}{3}\right)$，

$\therefore \overrightarrow{AF} \cdot \overrightarrow{EF} = -2 + t\left(t + \frac{1}{3}\right) = t^2 + \frac{1}{3}t - 2$ ，

当$t = -\frac{1}{6}$时，$\left(\overrightarrow{AF} \cdot \overrightarrow{EF}\right)_{\min} = \frac{1}{36} - \frac{1}{18} - 2 = -\frac{73}{36}$.

故选D .

方法二：



$\overrightarrow{AE} = \overrightarrow{AB} + \overrightarrow{BE}$

$= \overrightarrow{AB} + \frac{2}{3}\overrightarrow{BC}$

$= \overrightarrow{AB} + \frac{2}{3}\overrightarrow{AD}$.

$\overrightarrow{BD} = \overrightarrow{AD} - \overrightarrow{AB}$.

$\because \overrightarrow{AE} \cdot \overrightarrow{BD} = -\frac{2}{3}$ ，

$\left(\overrightarrow{AB} + \frac{2}{3}\overrightarrow{AD}\right)\left(\overrightarrow{AD} - \overrightarrow{AB}\right) = -\frac{2}{3}$ ，

$\therefore \frac{2}{3}\overrightarrow{AD}^2 - \overrightarrow{AB}^2 + \frac{1}{3}\overrightarrow{AB} \cdot \overrightarrow{AD} = -\frac{2}{3}$ ，

$\frac{2}{3} \times \left|\overrightarrow{AD}\right|^2 - \left|\overrightarrow{AB}\right|^2 + \frac{1}{3}\left|\overrightarrow{AB}\right| \cdot \left|\overrightarrow{AD}\right| \cos A = -\frac{2}{3}$ ，

$\frac{2}{3} \times 4 - 4 + \frac{1}{3} \times 2 \times 2\cos A = -\frac{2}{3}$ ，

$8 - 12 + 4\cos A = -2$ ，

$\cos A = \frac{1}{2}$ ，

$\therefore \angle A = 60°$.

设$\overrightarrow{BF} = \lambda\overrightarrow{BD}$，$\lambda \in [0,1]$ ，

$\overrightarrow{AF} = \overrightarrow{AB} + \overrightarrow{BF}$

$= \overrightarrow{AB} + \lambda\overrightarrow{BD}$

$= \overrightarrow{AB} + \lambda\left(\overrightarrow{AD} - \overrightarrow{AB}\right)$

$= (1-\lambda)\overrightarrow{AB} + \lambda\overrightarrow{AD}$.

$\overrightarrow{EF} = \overrightarrow{EB} + \overrightarrow{BF}$

$= \frac{2}{3}\overrightarrow{CB} + \lambda\overrightarrow{BD}$

$= -\frac{2}{3}\overrightarrow{AD} + \lambda\left(\overrightarrow{AD} - \overrightarrow{AB}\right)$

$= \left(\lambda - \frac{2}{3}\right)\overrightarrow{AD} - \lambda\overrightarrow{AB}$.

$\overrightarrow{AF} \cdot \overrightarrow{EF}$

$$=\left[(1-\lambda)\overrightarrow{AB}+\lambda\overrightarrow{AD}\right]\left[\left(\lambda-\frac{2}{3}\right)\overrightarrow{AD}-\lambda\overrightarrow{AB}\right]$$
$$=-\lambda(1-\lambda)\overrightarrow{AB}^2+\lambda\left(\lambda-\frac{2}{3}\right)\overrightarrow{AD}^2+\left[(1-\lambda)\left(\lambda-\frac{2}{3}\right)-\lambda^2\right]\overrightarrow{AB}\cdot\overrightarrow{AD}$$
$$=-4\lambda(1-\lambda)+4\lambda\left(\lambda-\frac{2}{3}\right)+\left[\lambda-\frac{2}{3}-\lambda^2+\frac{2}{3}\lambda-\lambda^2\right]\times2\times2\times\frac{1}{2}$$
$$=-4\lambda+4\lambda^2+4\lambda^2-\frac{8}{3}\lambda+2\left(-2\lambda^2+\frac{5}{3}\lambda-\frac{2}{3}\right)$$
$$=-4\lambda+8\lambda^2-\frac{8}{3}\lambda-4\lambda^2+\frac{10}{3}\lambda-\frac{4}{3}$$
$$=4\lambda^2-\frac{10}{3}\lambda-\frac{4}{3}$$
$$=4\left(\lambda-\frac{5}{12}\right)^2-4\times\frac{25}{144}-\frac{4}{3}$$
$$\geqslant-\frac{25}{36}-\frac{4}{3}=-\frac{73}{36}\ .$$

故选D．