02三角恒等变换综合

二倍角公式

（1）$\sin 2\alpha = 2\sin\alpha\cos\alpha$;

变形式$\sin\alpha\cos\alpha = \frac{1}{2}\sin 2\alpha$．

（2）$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 1 - 2\sin^2\alpha = 2\cos^2\alpha - 1$;变形式$\cos^2\alpha = \frac{\cos 2\alpha + 1}{2}$；

$\sin^2 x = \frac{1-\cos 2\alpha}{2}$．

（3）$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$．

直接利用二倍角公式

1　已知$a \in \left(\frac{\pi}{2}, \pi\right)$，$\sin\alpha = \frac{\sqrt{5}}{5}$，则$\tan 2\alpha =$ ______ ．

答案　$-\frac{4}{3}$

解析　$\because \sin a = \frac{\sqrt{5}}{5}$，$a \in \left(\frac{\pi}{2}, \pi\right)$，

$\therefore \cos a = -\sqrt{1-\sin^2 a} = -\frac{2\sqrt{5}}{5}$．

$\therefore \tan a = \frac{\sin a}{\cos a} = -\frac{1}{2}$，$\therefore \tan 2a = \frac{2\tan a}{1-\tan^2 a} = \frac{-1}{1-\frac{1}{4}} = -\frac{4}{3}$．

故答案为$-\frac{4}{3}$．

2　已知$\tan\theta = 2$，则$\cos 2\theta =$ ______ ．

答案　$-\frac{3}{5}$

解析　因为$\tan\theta = 2$，

所以$\cos 2\theta$

$= \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta}$

$= \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$=\dfrac{1-2^2}{1+2^2}$

$=-\dfrac{3}{5}$.

3 若$\theta\in\left[\dfrac{\pi}{4},\dfrac{\pi}{2}\right]$，$\cos 2\theta=-\dfrac{1}{8}$，则$\sin\theta=$（　）.

A. $\dfrac{3}{5}$

B. $\dfrac{3}{4}$

C. $\dfrac{\sqrt{7}}{4}$

D. $\dfrac{4}{5}$

答案 B

解析 $\because\cos 2\theta=-\dfrac{1}{8}=1-2\sin^2\theta$，

$\therefore\sin^2\theta=\dfrac{9}{16}$，

$\because\theta\in\left[\dfrac{\pi}{4},\dfrac{\pi}{2}\right]$，

$\therefore\sin\theta=\sqrt{\dfrac{9}{16}}=\dfrac{3}{4}$.

故选B .

4 已知$\cos\alpha=-\dfrac{1}{3}$，α为第二象限角，则$\tan\dfrac{\alpha}{2}$的值为（　）.

A. $\sqrt{2}$

B. $\dfrac{\sqrt{2}}{2}$

C. $-\sqrt{2}$

D. $2\sqrt{2}$

答案 A

解析 $\tan\dfrac{\alpha}{2}=\dfrac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}=\dfrac{\sin\alpha}{2\cos^2\frac{\alpha}{2}}$

$=\dfrac{\sqrt{1-\cos^2\alpha}}{1+\cos\alpha}$

$=\dfrac{\sqrt{1-\left(-\frac{1}{3}\right)^2}}{1-\frac{1}{3}}=\dfrac{\frac{2\sqrt{2}}{3}}{\frac{2}{3}}=\sqrt{2}$.

故选A .

5 若$\sin\left(\dfrac{\pi}{3}-x\right)=\dfrac{4}{5}$，则$\cos\left(\dfrac{2\pi}{3}-2x\right)$的值为（　）.

A. $-\dfrac{24}{25}$　　B. $\dfrac{24}{25}$　　C. $\dfrac{7}{25}$　　D. $-\dfrac{7}{25}$

答案　D

解析　$\sin\left(\dfrac{\pi}{3}-x\right)=\dfrac{4}{5}$，

$\cos\left(\dfrac{2}{3}\pi-2x\right)=1-2\sin^2\left(\dfrac{\pi}{3}-x\right)$，

$=1-2\cdot\dfrac{16}{25}$，

$=-\dfrac{7}{25}$．

故选：D

6　设α为锐角，若$\cos\left(\alpha+\dfrac{\pi}{6}\right)=\dfrac{4}{5}$，则$\sin\left(2\alpha+\dfrac{\pi}{12}\right)$的值为_____．

答案　$\dfrac{17\sqrt{2}}{50}$

解析　方法一：$\because\alpha$为锐角且$\cos\left(\alpha+\dfrac{\pi}{6}\right)=\dfrac{4}{5}$，

$\therefore\sin\left(\alpha+\dfrac{\pi}{6}\right)=\dfrac{3}{5}$．

$\therefore\sin\left(2\alpha+\dfrac{\pi}{12}\right)=\sin\left[2\left(\alpha+\dfrac{\pi}{6}\right)-\dfrac{\pi}{4}\right]$

$=\sin 2\left(\alpha+\dfrac{\pi}{6}\right)\cos\dfrac{\pi}{4}-\cos 2\left(\alpha+\dfrac{\pi}{6}\right)\sin\dfrac{\pi}{4}$

$=\sqrt{2}\sin\left(\alpha+\dfrac{\pi}{6}\right)\cos\left(\alpha+\dfrac{\pi}{6}\right)-\dfrac{\sqrt{2}}{2}\left[2\cos^2\left(\alpha+\dfrac{\pi}{6}\right)-1\right]$

$=\sqrt{2}\times\dfrac{3}{5}\times\dfrac{4}{5}-\dfrac{\sqrt{2}}{2}\left[2\times\left(\dfrac{4}{5}\right)^2-1\right]$

$=\dfrac{12\sqrt{2}}{25}-\dfrac{7\sqrt{2}}{50}$

$=\dfrac{17\sqrt{2}}{50}$．

方法二：$\because\alpha$为锐角，$\cos\left(\alpha+\dfrac{\pi}{6}\right)=\dfrac{4}{5}$，

$\therefore 0<\alpha+\dfrac{\pi}{6}<\dfrac{\pi}{2}$，$\sin\left(\alpha+\dfrac{\pi}{6}\right)=\dfrac{3}{5}$，

$\therefore\sin\left(2\alpha+\dfrac{\pi}{3}\right)=2\sin\left(\alpha+\dfrac{\pi}{6}\right)\cos\left(\alpha+\dfrac{\pi}{6}\right)=\dfrac{24}{25}$，

$\cos\left(2\alpha+\dfrac{\pi}{3}\right)=2\cos^2\left(\alpha+\dfrac{\pi}{6}\right)-1=\dfrac{7}{25}$，

$\therefore\sin\left(2\alpha+\dfrac{\pi}{12}\right)=\sin\left(2\alpha+\dfrac{\pi}{3}-\dfrac{\pi}{4}\right)=\dfrac{\sqrt{2}}{2}\left[\sin\left(2\alpha+\dfrac{\pi}{3}\right)-\cos\left(2\alpha+\dfrac{\pi}{3}\right)\right]=\dfrac{17\sqrt{2}}{50}$．

7 若$\alpha\in\left(\frac{\pi}{2},\pi\right)$，且$3\cos 2\alpha=\sin\left(\frac{\pi}{4}-\alpha\right)$，则$\sin 2\alpha$的值为（　　）．

A. $\frac{1}{18}$　　B. $-\frac{1}{18}$　　C. $\frac{17}{18}$　　D. $-\frac{17}{18}$

答案　D

解析　方法一：$\because\alpha\in\left(\frac{\pi}{2},\pi\right)$，$\therefore\sin\alpha>0$，$\cos\alpha<0$，

$\because 3\cos 2\alpha=\sin\left(\frac{\pi}{4}-\alpha\right)$，

$\therefore 3\left(\cos^2\alpha-\sin^2\alpha\right)=\frac{\sqrt{2}}{2}(\cos\alpha-\sin\alpha)$，

$\therefore\cos\alpha+\sin\alpha=\frac{\sqrt{2}}{6}$，

$\therefore$两边平方，可得：$1+2\sin\alpha\cos\alpha=\frac{1}{18}$，

$\therefore\sin 2\alpha=2\sin\alpha\cos\alpha=-\frac{17}{18}$．

故选：D．

方法二：$\cos 2\alpha=\sin\left(\frac{\pi}{2}-2a\right)=\sin\left[2\left(\frac{\pi}{4}-\alpha\right)\right]=2\sin\left(\frac{\pi}{4}-\alpha\right)\cos\left(\frac{\pi}{4}-\alpha\right)$代入原式，得

$6\sin\left(\frac{\pi}{4}-\alpha\right)\cos\left(\frac{\pi}{4}-\alpha\right)=\sin\left(\frac{\pi}{4}-\alpha\right)$，

$\because\alpha\in\left(\frac{\pi}{2},\pi\right)$，

$\therefore\frac{\pi}{4}-\alpha\neq 0$，

$\therefore\sin\alpha=\left(\frac{\pi}{4}-a\right)\neq 0$，

$\therefore\cos\left(\frac{\pi}{4}-\alpha\right)=\frac{1}{6}$，

$\therefore\sin 2\alpha=\cos\left(\frac{\pi}{2}-2\alpha\right)=2\cos^2\left(\frac{\pi}{4}-\alpha\right)=-\frac{17}{18}$．

故选D．

方法三：$\because 3\cos 2\alpha=\sin\left(\frac{\pi}{4}-\alpha\right)$，

$\therefore 3\left(\cos^2\alpha-\sin^2\alpha\right)=\frac{\sqrt{2}}{2}(\cos\alpha-\sin\alpha)$，

$\because a\in\left(\frac{\pi}{2},\pi\right)$，

$\therefore\cos\alpha\neq\sin\alpha$，

$\therefore 3(\cos\alpha+\sin\alpha)=\frac{\sqrt{2}}{2}$，即$\sin\alpha+\cos\alpha=\frac{\sqrt{2}}{6}$，

$\therefore(\sin\alpha+\cos\alpha)^2=2\sin\alpha\cdot\cos\alpha+1=\frac{2}{36}=\frac{1}{18}$，

$\therefore\sin 2\alpha=\frac{1}{18}-1=-\frac{17}{18}$．

故选D．

二倍角公式与诱导公式结合

8 设$\sin\left(\frac{\pi}{4}+\theta\right)=\frac{1}{3}$，则$\sin 2\theta=$（　）．

A. $-\frac{7}{9}$　　B. $-\frac{1}{9}$　　C. $\frac{1}{9}$　　D. $\frac{7}{9}$

答案 A

解析 由$\sin\left(\frac{\pi}{4}+\theta\right)=\sin\frac{\pi}{4}\cos\theta+\cos\frac{\pi}{4}\sin\theta=\frac{\sqrt{2}}{2}(\sin\theta+\cos\theta)=\frac{1}{3}$，

两边平方得：$1+2\sin\theta\cos\theta=\frac{2}{9}$，即$2\sin\theta\cos\theta=-\frac{7}{9}$，

则$\sin 2\theta=2\sin\theta\cos\theta=-\frac{7}{9}$．

故选A．

9 若$\cos\left(\frac{\pi}{4}-\alpha\right)=\frac{3}{5}$，则$\sin 2\alpha=$（　）．

A. $\frac{7}{25}$　　B. $-\frac{7}{25}$　　C. $-\frac{1}{5}$　　D. $\frac{1}{5}$

答案 B

解析 $\because\cos\left(\frac{\pi}{4}-\alpha\right)=\frac{3}{5}$，

$\therefore\sin 2\alpha=\cos\left(\frac{\pi}{2}-2\alpha\right)=2\cos^2\left(\frac{\pi}{4}-\alpha\right)-1$

$=\frac{18}{25}-1$

$=-\frac{7}{25}$，

$\therefore\cos\left(\frac{\pi}{2}-2\alpha\right)=\sin 2\alpha=-\frac{7}{25}$．

故选B．

10 若$\sin\left(\frac{\pi}{12}-\alpha\right)=\frac{\sqrt{3}}{2}$，则$\sin\left(2\alpha-\frac{2\pi}{3}\right)=$（　）．

A. $\frac{1}{2}$　　B. $-\frac{1}{2}$

C. $\frac{\sqrt{3}}{2}$　　D. $-\frac{\sqrt{3}}{2}$

答案 A

解析　因为$\sin\left(\frac{\pi}{12}-\alpha\right)=\frac{\sqrt{3}}{2}$，

所以$\cos\left(\frac{\pi}{6}-2\alpha\right)=1-2\times\left(\frac{\sqrt{3}}{2}\right)^2=-\frac{1}{2}$，

所以$\sin\left(2\alpha-\frac{2\pi}{3}\right)=\sin\left[\left(2\alpha-\frac{\pi}{6}\right)-\frac{\pi}{2}\right]=-\cos\left(2\alpha-\frac{\pi}{6}\right)=-\cos\left(\frac{\pi}{6}-2\alpha\right)=\frac{1}{2}$．

故选$\mathbf{A}$．

11　已知α满足$\cos 2\alpha=\frac{7}{9}$，则$\cos\left(\frac{\pi}{4}+\alpha\right)\cos\left(\frac{\pi}{4}-\alpha\right)=$（　）．

A. $\frac{7}{18}$　　B. $\frac{25}{18}$　　C. $-\frac{7}{18}$　　D. $-\frac{25}{18}$

答案　A

解析　$\because\alpha$满足$\cos 2\alpha=\frac{7}{9}$，

则 $\cos\left(\frac{\pi}{4}+\alpha\right)\cos\left(\frac{\pi}{4}-\alpha\right)$

$=\cos\left(\frac{\pi}{4}+\alpha\right)\cos\left[\frac{\pi}{2}-\left(\frac{\pi}{4}+\alpha\right)\right]$

$=\cos\left(\frac{\pi}{4}+\alpha\right)\sin\left(\frac{\pi}{4}+\alpha\right)$

$=\frac{1}{2}\sin\left(\frac{\pi}{2}+2\alpha\right)$

$=\frac{1}{2}\cos 2\alpha$

$=\frac{7}{18}$．

故选：$\mathbf{A}$．

12　已知θ是第四象限角，且$\cos\theta=\frac{4}{5}$，那么$\frac{\sin\left(\theta+\frac{\pi}{4}\right)}{\cos(2\theta-6\pi)}$的值为______．

答案　$\frac{5\sqrt{2}}{14}$

解析　依题意，有：$\sin\theta=-\frac{3}{5}$，

$\frac{\sin\left(\theta+\frac{\pi}{4}\right)}{\cos(2\theta-6\pi)}=\frac{\sin\theta\cos\frac{\pi}{4}+\cos\theta\sin\frac{\pi}{4}}{\cos 2\theta}=\frac{-\frac{3}{5}\times\frac{\sqrt{2}}{2}+\frac{4}{5}\times\frac{\sqrt{2}}{2}}{2\times\left(\frac{4}{5}\right)^2-1}=\frac{5\sqrt{2}}{14}$，

故答案为：$\frac{5\sqrt{2}}{14}$．

13 已知$\sin 2\alpha=\dfrac{2}{3}$，则$\cos^2\left(\alpha+\dfrac{\pi}{4}\right)=$（　）．

A. $\dfrac{1}{6}$　　B. $\dfrac{1}{3}$　　C. $\dfrac{1}{2}$　　D. $\dfrac{2}{3}$

答案 A

解析 $\because\sin 2\alpha=\dfrac{2}{3}$，

$\therefore\cos^2\left(\alpha+\dfrac{\pi}{4}\right)=\dfrac{1}{2}\left[1+\cos\left(2\alpha+\dfrac{\pi}{2}\right)\right]$

$=\dfrac{1}{2}(1-\sin 2\alpha)=\dfrac{1}{2}\times\left(1-\dfrac{2}{3}\right)=\dfrac{1}{6}$．

故选$\mathbf{A}$．

辅助角公式

$$y=a\sin\alpha+b\cos\alpha=\sqrt{a^2+b^2}\left(\frac{a}{\sqrt{a^2+b^2}}\sin\alpha+\frac{b}{\sqrt{a^2+b^2}}\cos\alpha\right)=\sqrt{a^2+b^2}\sin(\alpha+\varphi)，$$

其中φ所在的象限由a、b的符号确定，φ角的值由$\tan\varphi=\dfrac{b}{a}$确定．

14 已知$\cos\left(\alpha-\dfrac{\pi}{6}\right)+\sin\alpha=\dfrac{4}{5}\sqrt{3}$，则$\sin\left(\alpha+\dfrac{7}{6}\pi\right)$的值是（　）．

A. $\dfrac{4}{5}$　　B. $-\dfrac{4}{5}$

C. $\dfrac{2}{3}\sqrt{3}$　　D. $-\dfrac{2}{3}\sqrt{3}$

答案 B

解析 $\cos\left(\alpha-\dfrac{\pi}{6}\right)+\sin\alpha$

$=\dfrac{\sqrt{3}}{2}\cos\alpha+\dfrac{1}{2}\sin\alpha+\sin\alpha$

$=\dfrac{\sqrt{3}}{2}\cos\alpha+\dfrac{3}{2}\sin\alpha$

$=\dfrac{4}{5}\sqrt{3}$．

$\therefore\dfrac{1}{2}\cos\alpha+\dfrac{\sqrt{3}}{2}\sin\alpha=\dfrac{4}{5}$，

$\therefore\sin\left(\alpha+\dfrac{\pi}{6}\right)=\dfrac{4}{5}$，

$\sin\left(\alpha+\dfrac{7\pi}{6}\right)$

$=\sin\left(\alpha+\dfrac{\pi}{6}+\pi\right)$

$=-\sin\left(\alpha+\frac{\pi}{6}\right)$

$=-\frac{4}{5}$.

故选B .

15 已知$\sin\theta+\sin\left(\theta+\frac{\pi}{3}\right)=1$，则$\sin\left(\theta+\frac{\pi}{6}\right)=$（　）.

A. $\frac{1}{2}$

B. $\frac{\sqrt{3}}{3}$

C. $\frac{2}{3}$

D. $\frac{\sqrt{2}}{2}$

答案 B

解析 由$\sin\theta+\sin\left(\theta+\frac{\pi}{3}\right)=1$，

可得：$\frac{3}{2}\sin\theta+\frac{\sqrt{3}}{2}\cos\theta=1$，

所以有$\sqrt{3}\sin\left(\theta+\frac{\pi}{6}\right)=1$，

故 $\sin\left(\theta+\frac{\pi}{6}\right)=\frac{\sqrt{3}}{3}$.

故选：B .

16 若$3\cos\alpha-\sqrt{3}\sin\alpha=A\sin(\alpha+\varphi)$，则$A$，$\varphi$的值分别为（　）.

A. $2\sqrt{3},-\frac{\pi}{3}$

B. $-2\sqrt{3},\frac{\pi}{6}$

C. $2\sqrt{3},\frac{2\pi}{3}$

D. $2\sqrt{3},-\frac{\pi}{6}$

答案 C

解析 $3\cos\alpha-\sqrt{3}\sin\alpha$

$=2\sqrt{3}\left(\frac{\sqrt{3}\cos\alpha}{2}-\frac{\sin\alpha}{2}\right)$

$=2\sqrt{3}\left(\sin\frac{\pi}{3}\cos\alpha-\cos\frac{\pi}{3}\sin\alpha\right)$

$=2\sqrt{3}\left(\sin\frac{2\pi}{3}\cos\alpha+\cos\frac{2\pi}{3}\sin\alpha\right)$

$=2\sqrt{3}\sin\left(\alpha+\frac{2\pi}{3}\right)$.

故选C .

17 计算：$\dfrac{1}{\sin 15^\circ}+\dfrac{1}{\cos 15^\circ}=$ ______ .

答案 $2\sqrt{6}$

解析 $\dfrac{1}{\sin 15^\circ}+\dfrac{1}{\cos 15^\circ}=\dfrac{\sin 15^\circ+\cos 15^\circ}{\sin 15^\circ\cos 15^\circ}$

$=\dfrac{\sqrt{2}\sin(15^\circ+45^\circ)}{\dfrac{1}{2}\sin 30^\circ}$

$=\dfrac{\sqrt{2}\sin 60^\circ}{\dfrac{1}{2}\sin 30^\circ}$

$=2\sqrt{6}$.

故答案为：$2\sqrt{6}$.

综合应用

18 若$\dfrac{\cos 2\alpha}{\sin\left(\alpha-\dfrac{\pi}{4}\right)}=-\dfrac{\sqrt{2}}{2}$，则$\sin\alpha+\cos\alpha=$ ______ .

答案 $\dfrac{1}{2}$

解析 $\because\dfrac{\cos 2\alpha}{\sin\left(\alpha-\dfrac{\pi}{4}\right)}=\dfrac{\cos^2\alpha-\sin^2\alpha}{\dfrac{\sqrt{2}}{2}\sin\alpha-\dfrac{\sqrt{2}}{2}\cos\alpha}$

$=\dfrac{(\cos\alpha-\sin\alpha)(\cos\alpha+\sin\alpha)}{\dfrac{\sqrt{2}}{2}(\sin\alpha-\cos\alpha)}$

$=-\dfrac{\sqrt{2}}{2}$,

$\therefore\cos\alpha+\sin\alpha=\dfrac{1}{2}$.

19 计算$\dfrac{\sin 110^\circ\sin 20^\circ}{\cos^2 155^\circ-\sin^2 155^\circ}$的值为 ______ .

答案 $\dfrac{1}{2}$

解析 $\dfrac{\sin 110^\circ \sin 20^\circ}{\cos^2 155^\circ - \sin^2 155^\circ}$

$=\dfrac{\sin(90^\circ+20^\circ)\sin 20^\circ}{\cos 310^\circ}$，

$=\dfrac{\cos 20^\circ \sin 20^\circ}{\cos(270^\circ+40^\circ)}$，

$=\dfrac{\sin 40^\circ}{2\sin 40^\circ}$，

$=\dfrac{1}{2}$．

20 已知$\tan\left(\alpha+\dfrac{\pi}{4}\right)=\dfrac{1}{2}$，且$-\dfrac{\pi}{2}<\alpha<0$，则$\dfrac{2\sin^2\alpha+\sin 2\alpha}{\cos\left(\alpha-\dfrac{\pi}{4}\right)}=$______．

答案 $\dfrac{-2\sqrt{5}}{5}$

解析 $\because\tan\left(\alpha+\dfrac{\pi}{4}\right)=\dfrac{1}{2}=\dfrac{1+\tan\alpha}{1-\tan\alpha}$，

$\therefore\tan\alpha=-\dfrac{1}{3}$．

再由$\tan\alpha=\dfrac{\sin\alpha}{\cos\alpha}$，$\sin^2\alpha+\cos^2\alpha=1$，$-\dfrac{\pi}{2}<\alpha<0$，

可得$\sin\alpha=-\dfrac{\sqrt{10}}{10}$．

故$\dfrac{2\sin^2\alpha+\sin 2\alpha}{\cos\left(\alpha-\dfrac{\pi}{4}\right)}$

$=\dfrac{2\sin\alpha(\sin\alpha+\cos\alpha)}{\cos\left(\dfrac{\pi}{4}-\alpha\right)}$

$=\dfrac{2\sin\alpha(\sin\alpha+\cos\alpha)}{\dfrac{\sqrt{2}}{2}(\cos\alpha+\sin\alpha)}$

$=2\sqrt{2}\sin\alpha$

$=2\sqrt{2}\times\left(-\dfrac{\sqrt{10}}{10}\right)$

$=\dfrac{-2\sqrt{5}}{5}$，

故答案为$\dfrac{-2\sqrt{5}}{5}$．

21 已知$\alpha\in\left(0,\dfrac{\pi}{2}\right)$，且$2\sin^2\alpha-\sin\alpha\cdot\cos\alpha-3\cos^2\alpha=0$，则$\dfrac{\sin\left(\alpha+\dfrac{\pi}{4}\right)}{\sin 2\alpha+\cos 2\alpha+1}=$______．

答案 $\dfrac{\sqrt{26}}{8}$

解析 $\because\alpha\in\left(0,\dfrac{\pi}{2}\right)$，且$2\sin^2\alpha-\sin\alpha\cdot\cos\alpha-3\cos^2\alpha=0$，则$(2\sin\alpha-3\cos\alpha)\cdot(\sin\alpha+\cos\alpha)=0$，

$\therefore 2\sin\alpha=3\cos\alpha$，又$\sin^2\alpha+\cos^2\alpha=1$，

$\therefore\cos\alpha=\dfrac{2}{\sqrt{13}}$，$\sin\alpha=\dfrac{3}{\sqrt{13}}$，$\therefore\dfrac{\sin\left(\alpha+\dfrac{\pi}{4}\right)}{\sin 2\alpha+\cos 2\alpha+1}$

$=\dfrac{\dfrac{\sqrt{2}}{2}(\sin\alpha+\cos\alpha)}{(\sin\alpha+\cos\alpha)^2+(\cos^2\alpha-\sin^2\alpha)}=\dfrac{\sqrt{26}}{8}$.

22 已知$\tan\left(\dfrac{\pi}{4}+\alpha\right)=\dfrac{1}{2}$，则$\dfrac{\sin(2\alpha+2\pi)-\sin^2\left(\dfrac{\pi}{2}-\alpha\right)}{1-\cos(\pi-2\alpha)+\sin^2\alpha}$的值为______ .

答案 $-\dfrac{15}{19}$

解析 $\because\tan\left(\dfrac{\pi}{4}+\alpha\right)=\dfrac{1}{2}$，

$\therefore\tan\left(\dfrac{\pi}{4}+\alpha\right)=\dfrac{\tan\frac{\pi}{4}+\tan\alpha}{1-\tan\frac{\pi}{4}\tan\alpha}=\dfrac{1}{2}$，

解得$\tan\alpha=-\dfrac{1}{3}$，

$\therefore\dfrac{\sin(2\alpha+2\pi)-\sin^2\left(\frac{\pi}{2}-\alpha\right)}{1-\cos(\pi-2\alpha)+\sin^2\alpha}$

$=\dfrac{\sin 2\alpha-\cos^2\alpha}{1+\cos 2\alpha+\sin^2\alpha}$

$=\dfrac{2\sin\alpha\cos\alpha-\cos^2\alpha}{\sin^2\alpha+2\cos^2\alpha}$

$=\dfrac{2\tan\alpha-1}{\tan^2\alpha+2}$，

将$\tan\alpha=-\dfrac{1}{3}$代入得原式$=-\dfrac{15}{19}$.