# 01三角函数定义与恒等变换练习题

## 1. 任意角与弧度制

1 已知扇形的周长为$8\mathrm{cm}$，圆心角为$2$，则扇形的面积为（　）．

A. $1$　　B. $2$　　C. $4$　　D. $5$

答案 C

解析 设扇形的半径为$r$，弧长为$l$，

则扇形的周长为$l+2r=8$，

$\therefore$弧长为：$\alpha r=2r$，

$\therefore r=2$，

根据扇形的面积公式，

得$S=\frac{1}{2}\alpha r^2=4$．

故选$C$．

2 已知扇形的周长为$10\mathrm{cm}$，面积为$4\mathrm{cm}^2$，则扇形圆心角的弧度数为______．

答案 $\frac{1}{2}$

解析 设扇形弧长为$l$，半径为$r$，

则$l+2r=10$，

$\because \frac{1}{2}(10-2r)r=4\Rightarrow r=1$（舍）或$4$，

$\therefore l=2$，$\alpha=\frac{l}{r}=\frac{1}{2}$．

3 已知圆锥的表面积为$9\pi\mathrm{cm}^2$，且它的侧面展开图是一个半圆，则圆锥的底面半径为（　）．

A. $\frac{3\sqrt{2}}{2}\mathrm{cm}$　　B. $3\sqrt{2}\mathrm{cm}$

C. $\sqrt{3}\text{cm}$

D. $2\sqrt{3}\text{cm}$

答案 C

解析 设圆锥的底面半径长为$r$，圆锥的母线长为$l$，则由$\pi l=2\pi r$，得$l=2r$，而
$S=\pi r^2+\pi r\cdot 2r=3\pi r^2=9\pi$，故$r^2=3$，解得$r=\sqrt{3}\text{cm}$．

## 2. 三角函数的定义

4 已知$\sin\theta+\cos\theta=\frac{7}{5}$，$\theta\in\left(0,\frac{\pi}{4}\right)$，则$\sin\theta-\cos\theta$的值为______．

答案 $-\frac{1}{5}$

解析 $\because\sin\theta+\cos\theta=\frac{7}{5}$，

$\therefore(\sin\theta+\cos\theta)^2=\frac{49}{25}$，

即$1+2\sin\theta\cdot\cos\theta=\frac{49}{25}$，

$2\sin\theta\cdot\cos\theta=\frac{24}{25}$，

$\therefore(\sin\theta-\cos\theta)^2=1-2\sin\theta\cdot\cos\theta=\frac{1}{25}$，

$\because\theta\in\left(0,\frac{\pi}{4}\right)$，

$\therefore\cos\theta>\sin\theta$，

$\therefore\sin\theta-\cos\theta=-\frac{1}{5}$．

5 已知$\sin\alpha-\cos\alpha=\sqrt{2}$，$\alpha\in(0,\pi)$，则$\tan\alpha=$（　）．

A. $-1$

B. $-\frac{\sqrt{2}}{2}$

C. $\frac{\sqrt{2}}{2}$

D. $1$

答案 A

解析

方法一：由$\begin{cases}1\sin\alpha-\cos\alpha=\sqrt{2}\\ \sin^2\alpha+\cos^2\alpha=1\end{cases}$，得$2\cos^2\alpha+2\sqrt{2}\cos\alpha+1=0$，即$(\sqrt{2}\cos\alpha+1)^2=0$，∴ $\cos\alpha=-\dfrac{\sqrt{2}}{2}$．又$\alpha\in(0,\pi)$，$\therefore\alpha=\dfrac{3\pi}{4}$，$\therefore\tan\alpha=\tan\dfrac{3\pi}{4}=-1$．

方法二：因为$\sin\alpha-\cos\alpha=\sqrt{2}$，所以$(\sin\alpha-\cos\alpha)^2=2$，所以$\sin 2\alpha=-1$，因为$\alpha\in(0,\pi)$，$2\alpha\in(0,2\pi)$，所以$2\alpha=\dfrac{3\pi}{2}$，所以$\alpha=\dfrac{3\pi}{4}$．所以$\tan\alpha=-1$．

方法三：因为$\sin\alpha-\cos\alpha=\sqrt{2}$，所以$\sqrt{2}\sin\left(\alpha-\dfrac{\pi}{4}\right)=\sqrt{2}$，所以$\sin\left(\alpha-\dfrac{\pi}{4}\right)=1$，因为$\alpha\in(0,\pi)$，所以$\alpha=\dfrac{3\pi}{4}$，所以$\tan\alpha=-1$．

6 若$\tan\theta=3$，则$2\sin^2\theta-\sin\theta\cos\theta-\cos^2\theta=$______．

答案 $\dfrac{7}{5}$

解析 思路一：求解$\sin\theta$，$\cos\theta$

根据同角关系：$\tan\theta=\dfrac{\sin\theta}{\cos\theta}$

$\tan\theta=3\Rightarrow\sin\theta=3\cos\theta$

又因为$\sin^2\theta+\cos^2\theta=1$

所以$\sin\theta=\dfrac{3}{\sqrt{10}}$，$\cos\theta=\dfrac{1}{\sqrt{10}}$或$\sin\theta=\dfrac{-3}{\sqrt{10}}$，$\cos\theta=-\dfrac{1}{\sqrt{10}}$

所以$2\sin^2\theta-\sin\theta\cdot\cos\theta-\cos^2\theta=\dfrac{18}{10}-\dfrac{3}{10}-\dfrac{1}{10}=\dfrac{7}{5}$

思路二，切化弦

根据同角关系，$\tan\theta=\dfrac{\sin\theta}{\cos\theta}$

$\tan\theta=3\Rightarrow\sin\theta=3\cos\theta$

整个式子除以1，并把1用$\sin^2\theta+\cos^2\theta$表示，将问题中的$\sin\theta$用$\cos\theta$表示，所以

$2\sin^2\theta-\sin\theta\cdot\cos\theta-\cos^2\theta=\dfrac{18\cos^2\theta-3\cos^2\theta-\cos^2\theta}{9\cos^2\theta+\cos^2\theta}=\dfrac{7}{5}$

思路三：弦化切

根据同角关系$\tan\theta=\dfrac{\sin\theta}{\cos\theta}$

整个式子除以1，并把1转化$\sin^2\theta+\cos^2\theta$，将分子分母同除以$\cos^2\theta$

所以$2\sin^2\theta-\sin\theta\cdot\cos\theta-\cos^2\theta=\dfrac{2\sin^2\theta-\sin\theta\cos\theta-\cos^2\theta}{\sin^2\theta+\cos^2\theta}=\dfrac{2\tan^2\theta-\tan\theta-1}{\tan^2\theta+1}=\dfrac{7}{5}$．

## 3. 诱导公式

7 已知$\cos\left(\alpha+\frac{\pi}{6}\right)=-\frac{1}{3}$，则$\sin\left(\alpha-\frac{\pi}{3}\right)$的值为（　）．

A. $\frac{1}{3}$　　B. $-\frac{1}{3}$

C. $\frac{2\sqrt{3}}{3}$　　D. $-\frac{2\sqrt{3}}{3}$

答案　A

解析　$\because\cos\left(\alpha+\frac{\pi}{6}\right)=-\frac{1}{3}$，

$\therefore\sin\left(\alpha-\frac{\pi}{3}\right)=\sin\left[\left(\alpha+\frac{\pi}{6}\right)-\frac{\pi}{2}\right]=-\cos\left(\alpha+\frac{\pi}{6}\right)=\frac{1}{3}$．

故选$\mathbf{A}$．

8 化简：$\dfrac{\sin(2\pi+\alpha)\cos(\pi-\alpha)\cos\left(\frac{\pi}{2}-\alpha\right)\cos\left(\frac{7\pi}{2}-\alpha\right)}{\cos(-\pi-\alpha)\sin(-3\pi-\alpha)\sin(-\pi+\alpha)\sin\left(\frac{5\pi}{2}+\alpha\right)}=$______．

答案　$\tan\alpha$

解析　原式$=\dfrac{\sin\alpha\cdot(-\cos\alpha)\cdot\sin\alpha\cdot(-\sin\alpha)}{-\cos\alpha\cdot\sin\alpha\cdot(-\sin\alpha)\cdot\cos\alpha}=\tan\alpha$．

故答案为：$\tan\alpha$．

9 已知$f(\alpha)=\dfrac{\sin(\alpha-\frac{\pi}{2})\cos(\frac{3\pi}{2}-\alpha)\tan(\pi+\alpha)\cos(\frac{\pi}{2}+\alpha)}{\sin(2\pi-\alpha)\tan(-\alpha-\pi)\sin(-\alpha-\pi)}$．

(1) 化简$f(\alpha)$．

(2) 若$\alpha$是第三象限角，且$\cos(\alpha-\frac{3\pi}{2})=\frac{1}{5}$,求$f(\alpha)$．

答案　(1) $-\cos\alpha$．

(2) $\frac{2\sqrt{6}}{5}$．

解析　(1) $f(\alpha)=\dfrac{-\sin(\frac{\pi}{2}-\alpha)(-\sin\alpha)\tan\alpha(-\sin\alpha)}{\sin(-\alpha)(-\tan\alpha)[-\sin(\pi+\alpha)]}=\dfrac{-\cos\alpha(-\sin\alpha)\tan\alpha(-\sin\alpha)}{-\sin\alpha(-\tan\alpha)\sin\alpha}=-\cos\alpha$

．

故$f(\alpha)=-\cos\alpha$．

(2) 因为$\cos(\alpha-\frac{3\pi}{2})=\cos(\frac{3\pi}{2}-\alpha)=-\sin\alpha=\frac{1}{5}$，

所以$\sin\alpha=-\frac{1}{5}$．

又$\alpha$为第三象限角，

所以$\cos\alpha=-\sqrt{1-\sin^2\alpha}=-\frac{2\sqrt{6}}{5}$，

所$f(\alpha)=-\cos\alpha=\frac{2\sqrt{6}}{5}$，

故答案为：$\frac{2\sqrt{6}}{5}$．

10 已知函数$f(\alpha)=\dfrac{\sin\left(\frac{\pi}{2}-\alpha\right)\cos\left(\alpha-\frac{3\pi}{2}\right)\tan(2\pi-\alpha)}{\tan(\alpha+\pi)\sin(\alpha+\pi)}$．

(1) 化简$f(\alpha)$．

(2) 若$f\left(\alpha+\frac{\pi}{2}\right)=2f(\alpha)$，求$f(\alpha)\cdot f\left(\frac{\pi}{2}-\alpha\right)$的值．

答案 (1) $f(\alpha)=-\cos\alpha$．

(2) $f(\alpha)\cdot f\left(\frac{\pi}{2}-\alpha\right)=-\frac{2}{5}$．

解析 (1)

$$f(\alpha)=\frac{\sin\left(\frac{\pi}{2}-\alpha\right)\cdot\cos\left(\alpha-\frac{3\pi}{2}\right)\tan(2\pi-\alpha)}{\tan(\alpha+\pi)\sin(\alpha+\pi)}$$

$$=\frac{\cos\alpha\cdot(-\sin\alpha)(-\tan\alpha)}{\tan\alpha\cdot(-\sin\alpha)}$$

$=-\cos\alpha$．

(2) $f\left(\alpha+\frac{\pi}{2}\right)=2f(\alpha)$，

$-\cos\left(\alpha+\frac{\pi}{2}\right)=-2\cos\alpha$，

$\sin\alpha=-2\cos\alpha$，

$\tan\alpha=-2$．

$f(\alpha)\cdot f\left(\frac{\pi}{2}-\alpha\right)$

$=-\cos\alpha\cdot\left[-\cos\left(\frac{\pi}{2}-\alpha\right)\right]$

$=\cos\alpha\cdot\sin\alpha$

$$=\frac{\cos\alpha\cdot\sin\alpha}{\cos^2\alpha+\sin^2\alpha}$$

$$=\frac{\tan\alpha}{1+\tan^2\alpha}$$

$=-\frac{2}{5}$．

## 4. 和差角公式

11 $\cos 20^\circ \cos 50^\circ - \cos 110^\circ \sin 50^\circ = (\quad)$ .

A. $-\frac{\sqrt{3}}{2}$

B. $\frac{\sqrt{3}}{2}$

C. $-\frac{1}{2}$

D. $\frac{1}{2}$

答案 B

解析 $\cos 20^\circ \cos 50^\circ - \cos 110^\circ 50^\circ$

$= \cos 20^\circ \cos 50^\circ + \sin 20^\circ \cdot \sin 50^\circ$

$= \cos(50^\circ - 20^\circ)$

$= \cos 30^\circ$

$= \frac{\sqrt{3}}{2}$ .

故选B .

12 已知$\alpha$，$\beta$都是锐角，$\tan\alpha = 2$，$\tan\beta = \frac{1}{2}$，则$\alpha + \beta =$ ______ .

答案 $\frac{\pi}{2}$

解析 $\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$ ,

又$1 - \tan\alpha\tan\beta = 0$ ,

$0 < \alpha + \beta < \pi$ ,

故$\alpha + \beta = \frac{\pi}{2}$ .

13 已知$\alpha$，$\beta \in (-\frac{\pi}{2}, \frac{\pi}{2})$，$\sin\alpha = \frac{3}{5}$，$\sin\beta = -\frac{\sqrt{2}}{10}$，则$\cos(\alpha - \beta) =$ ______ .

答案 $\frac{\sqrt{2}}{2}$

解析 因为$\alpha$，$\beta\in(-\frac{\pi}{2},\frac{\pi}{2})$，所以$\cos\alpha=\frac{4}{5}$，$\cos\beta=\frac{7\sqrt{2}}{10}$，

$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta=\frac{4}{5}\cdot\frac{7\sqrt{2}}{10}+\frac{3}{5}\cdot(\frac{-\sqrt{2}}{10})=\frac{\sqrt{2}}{2}$．

14 已知$\tan(\alpha+\beta)=\frac{2}{5}$，$\tan\left(\beta-\frac{\pi}{4}\right)=\frac{1}{4}$，那么$\tan\left(\alpha+\frac{\pi}{4}\right)$的值是______．

答案 $\frac{3}{22}$

解析 因为$\tan(\alpha+\beta)=\frac{2}{5}$，$\tan\left(\beta-\frac{\pi}{4}\right)=\frac{1}{4}$，

所以$\tan\left(\alpha+\frac{\pi}{4}\right)=\tan\left[(\alpha+\beta)-\left(\beta-\frac{\pi}{4}\right)\right]$

$=\dfrac{\tan(\alpha+\beta)-\tan(\beta-\frac{\pi}{4})}{1+\tan(\alpha+\beta)\tan(\beta-\frac{\pi}{4})}$

$=\dfrac{\frac{2}{5}-\frac{1}{4}}{1+\frac{2}{5}\times\frac{1}{4}}=\dfrac{3}{22}$，

故答案为：$\frac{3}{22}$．

15 已知$\tan\alpha=\frac{\sqrt{2}}{2}$，$\tan(\alpha-\beta)=-\frac{\sqrt{2}}{2}$，则$\tan\beta=$______．

答案 $2\sqrt{2}$

解析 方法一：$\tan(\alpha-\beta)=\dfrac{\tan\alpha-\tan\beta}{1-\tan\alpha\cdot\tan\beta}=-\dfrac{\sqrt{2}}{2}$．

$\dfrac{\frac{\sqrt{2}}{2}-\tan\beta}{1+\frac{\sqrt{2}}{2}\tan\beta}-\dfrac{\sqrt{2}}{2}$，$\tan\beta=2\sqrt{2}$．

方法二：$\because\tan\alpha=\frac{\sqrt{2}}{2}$，$\tan(\alpha-\beta)=-\frac{\sqrt{2}}{2}$，

$\therefore\tan\beta=\tan[\alpha-(\alpha-\beta)]$

$=\dfrac{\tan\alpha-\tan(\alpha-\beta)}{1+\tan\alpha\cdot\tan(\alpha-\beta)}$

$=\dfrac{\frac{\sqrt{2}}{2}-\left(-\frac{\sqrt{2}}{2}\right)}{1+\frac{\sqrt{2}}{2}\cdot\left(-\frac{\sqrt{2}}{2}\right)}$

$=2\sqrt{2}$．

16 已知$\alpha,\beta\in\left(0,\frac{\pi}{2}\right)$，且满足$\sin\alpha=\frac{\sqrt{10}}{10}$，$\cos\beta=\frac{2\sqrt{5}}{5}$，则$\alpha+\beta$的值为（　）．

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. $\frac{3\pi}{4}$

D. $\frac{\pi}{4}$或$\frac{3\pi}{4}$

答案 A

解析 由$\alpha,\beta\in\left(0,\frac{\pi}{2}\right)$，$\sin\alpha=\frac{\sqrt{10}}{10}$，$\cos\beta=\frac{2\sqrt{5}}{5}$，

$\therefore\cos\alpha>0,\sin\beta>0$，

$\cos\alpha=\sqrt{1-\sin^2\alpha}=\sqrt{1-\left(\frac{\sqrt{10}}{10}\right)^2}=\frac{3\sqrt{10}}{10}$，

$\sin\alpha=\sqrt{1-\cos^2\alpha}=\sqrt{1-\left(\frac{2\sqrt{5}}{5}\right)^2}=\frac{\sqrt{5}}{5}$

$\therefore\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$=\frac{3\sqrt{10}}{10}\times\frac{2\sqrt{5}}{5}-\frac{\sqrt{10}}{10}\times\frac{\sqrt{5}}{5}=\frac{\sqrt{2}}{2}$，

由$\alpha,\beta\in\left(0,\frac{\pi}{2}\right)$可得$0<\alpha+\beta<\pi$，

$\therefore\alpha+\beta=\frac{\pi}{4}$．

故选：$\mathbf{A}$．

17 已知$\tan\alpha=2$，则$\frac{2\sin\alpha+3\cos\alpha}{\sin\alpha-\cos\alpha}=$______．

答案 $7$

解析 $\frac{2\sin\alpha+3\cos\alpha}{\sin\alpha-\cos\alpha}$

$=\frac{2\tan\alpha+3}{\tan\alpha-1}=7$．

故答案为：$7$．

18 若$0<\alpha<\frac{\pi}{2}$，$-\frac{\pi}{2}<\beta<0$，$\cos\left(\frac{\pi}{4}+\alpha\right)=\frac{1}{3}$，$\cos\left(\frac{\pi}{4}-\frac{\beta}{2}\right)=\frac{\sqrt{3}}{3}$，则$\cos\left(\alpha+\frac{\beta}{2}\right)=$（　）．

A. $\frac{\sqrt{3}}{3}$

B. $-\frac{\sqrt{3}}{3}$

C. $\frac{5\sqrt{3}}{9}$

D. $-\frac{\sqrt{6}}{9}$

答案　C

解析　若$-\frac{\pi}{2}<\beta<0<\alpha<\frac{\pi}{2}$，$\cos\left(\frac{\pi}{4}+\alpha\right)=\frac{1}{3}$，$\cos\left(\frac{\pi}{4}-\frac{\beta}{2}\right)=\frac{\sqrt{3}}{3}$，

$\therefore\sin\left(\frac{\pi}{4}+\alpha\right)=\frac{2\sqrt{2}}{3}$，$\sin\left(\frac{\pi}{4}-\frac{\beta}{2}\right)=\frac{\sqrt{6}}{3}$，

$\therefore\cos\left(\alpha+\frac{\beta}{2}\right)=\cos\left[\left(\frac{\pi}{4}+\alpha\right)-\left(\frac{\pi}{4}-\frac{\beta}{2}\right)\right]$

$=\cos\left(\frac{\pi}{4}+\alpha\right)\cos\left(\frac{\pi}{4}-\frac{\beta}{2}\right)+\sin\left(\frac{\pi}{4}+\alpha\right)\sin\left(\frac{\pi}{4}-\frac{\beta}{2}\right)$

$=\frac{1}{3}\times\frac{\sqrt{3}}{3}+\frac{2\sqrt{2}}{3}\times\frac{\sqrt{6}}{3}$

$=\frac{5\sqrt{3}}{9}$．

故选：C．

19　已知$\cos\alpha=\frac{1}{7}$，$\cos(\alpha+\beta)=-\frac{11}{14}$，且$\alpha\in\left(0,\frac{\pi}{2}\right)$，$\alpha+\beta\in\left(\frac{\pi}{2},\pi\right)$．

(1)　求$\tan 2\alpha$的值．

(2)　求$\beta$的值．

答案　(1)　$-\frac{8\sqrt{3}}{47}$．

(2)　$\frac{\pi}{3}$．

解析　(1)　$\because\cos\alpha=\frac{1}{7}$且$\alpha\in\left(0,\frac{\pi}{2}\right)$，

$\therefore\sin\alpha=\frac{4\sqrt{3}}{7}$，$\tan\alpha=4\sqrt{3}$，

$\tan 2\alpha=\frac{2\tan\alpha}{1-\tan^2\alpha}=\frac{8\sqrt{3}}{1-(4\sqrt{3})^2}=-\frac{8\sqrt{3}}{47}$．

(2)　$\because\cos(\alpha+\beta)=-\frac{11}{14}$且$\alpha+\beta\in\left(\frac{\pi}{2},\pi\right)$，

$\therefore sin(\alpha+\beta)=\sqrt{1-\cos^2(\alpha+\beta)}=\frac{5\sqrt{3}}{14}$，

$\therefore\cos\beta=\cos[(\alpha+\beta)-\alpha]$

$=\cos(\alpha+\beta)\cdot\cos\alpha+sin(\alpha+\beta)\cdot\sin\alpha$

$=-\frac{11}{14}\times\frac{1}{7}+\frac{5\sqrt{3}}{14}\times\frac{4\sqrt{3}}{7}$

$=\frac{1}{2}$，

$\because\alpha\in\left(0,\frac{\pi}{2}\right)$，$\alpha+\beta\in\left(\frac{\pi}{2},\pi\right)$，

$\therefore\beta\in(0,\pi)$，

$\therefore\beta=\frac{\pi}{3}$．

20 已知$3\sin(\pi-\alpha)+\cos(2\pi-\alpha)=0$．

(1) 求$\dfrac{\sin\alpha+\cos\alpha}{2\sin\alpha-\cos\alpha}$．

(2) 求$\dfrac{\sin 2\alpha+\cos^2\alpha}{2\cos 2\alpha+\sin 2\alpha+2}$．

答案 (1) $-\dfrac{2}{5}$．

(2) $\dfrac{1}{10}$．

解析 (1) 已知$3\sin(\pi-\alpha)+\cos(2\pi-\alpha)=0$．

可得：$3\sin\alpha+\cos\alpha=0$，即$\tan\alpha=-\dfrac{1}{3}$．

$$\frac{\sin\alpha+\cos\alpha}{2\sin\alpha-\cos\alpha}=\frac{\tan\alpha+1}{2\tan\alpha-1}=\frac{-\frac{1}{3}+1}{-\frac{1}{3}\times 2-1}=\frac{\frac{2}{3}}{-\frac{5}{3}}=-\frac{2}{5}.$$

(2) $$\frac{\sin 2\alpha+\cos^2\alpha}{2\cos 2\alpha+\sin 2\alpha+2}=\frac{2\sin\alpha\cos\alpha+\cos^2\alpha}{2\left(2\cos^2\alpha-1\right)+2\sin\alpha\cos\alpha}=\frac{2\tan\alpha-1}{4+2\tan\alpha}=\frac{1}{10}.$$