02三角恒等变换综合练习题

1. 二倍角公式

1 设$\sin\left(\frac{\pi}{4}+\theta\right)=\frac{1}{3}$，则$\sin 2\theta=$（　　）．

A. $-\frac{7}{9}$　　B. $-\frac{1}{9}$　　C. $\frac{1}{9}$　　D. $\frac{7}{9}$

答案　A

解析　方法一：由$\sin\left(\frac{\pi}{4}+\theta\right)=\sin\frac{\pi}{4}\cos\theta+\cos\frac{\pi}{4}\sin\theta=\frac{\sqrt{2}}{2}(\sin\theta+\cos\theta)=\frac{1}{3}$，

两边平方得：$1+2\sin\theta\cos\theta=\frac{2}{9}$，即$2\sin\theta\cos\theta=-\frac{7}{9}$，

则$\sin 2\theta=2\sin\theta\cos\theta=-\frac{7}{9}$．

故选A．

方法二：$\sin 2\theta=\sin\left[2\left(\frac{\pi}{4}+\theta\right)-\frac{\pi}{2}\right]=-\cos\left[2\left(\frac{\pi}{4}+\theta\right)\right]=-\left[1-2\sin^2\left(\frac{\pi}{4}+\theta\right)\right]=-\frac{7}{9}$．

故选A．

2 若$\sin\left(\frac{\pi}{2}+\theta\right)=\frac{3}{5}$，则$\cos 2\theta=$______．

答案　$-\frac{7}{25}$

解析　由$\sin\left(\frac{\pi}{2}+\theta\right)=\frac{3}{5}$可知，$\cos\theta=\frac{3}{5}$，

而$\cos 2\theta=2\cos^2\theta-1=2\times\left(\frac{3}{5}\right)^2-1=-\frac{7}{25}$．

故答案为：$-\frac{7}{25}$．

3 若$\theta\in[\frac{\pi}{4},\frac{\pi}{2}]$，$\sin 2\theta=\frac{3\sqrt{7}}{8}$，则$\sin\theta=$（　）．

A. $\frac{3}{5}$　　B. $\frac{4}{5}$

C. $\frac{\sqrt{7}}{4}$　　D. $\frac{3}{4}$

答案 D

解析 因为$\theta \in [\frac{\pi}{4},\frac{\pi}{2}]$，

所以$2\theta \in [\frac{\pi}{2},\pi]$，所以$\cos 2\theta \leqslant 0$，

所以$\cos 2\theta = -\sqrt{1-\sin^2 2\theta} = -\frac{1}{8}$．

又$\cos 2\theta = 1-2\sin^2\theta = -\frac{1}{8}$，

所以$\sin^2\theta = \frac{9}{16}$，

所以$\sin\theta = \frac{3}{4}$．

4 若$\tan\theta = -\frac{1}{3}$，则$\cos 2\theta =$（　）．

A. $-\frac{4}{5}$　　B. $-\frac{1}{5}$　　C. $\frac{1}{5}$　　D. $\frac{4}{5}$

答案 D

解析 方法一：$\cos 2\theta = \cos^2\theta - \sin^2\theta = \frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta+\sin^2\theta} = \frac{1-\tan^2\theta}{1+\tan^2\theta}$，

把$\tan\theta = -\frac{1}{3}$代入，原式$= \frac{4}{5}$．

故选D．

方法二：由$\tan\theta = -\frac{1}{3}$，可得$\sin\theta = \pm\frac{1}{\sqrt{10}}$，

因而$\cos 2\theta = 1-2\sin^2\theta = \frac{4}{5}$．

故选D．

5 已知α为第二象限角，$\sin\alpha+\cos\alpha = \frac{\sqrt{3}}{3}$，则$\cos 2\alpha =$（　）．

A. $-\frac{\sqrt{5}}{3}$　　B. $-\frac{\sqrt{5}}{9}$

C. $\frac{\sqrt{5}}{9}$　　D. $\frac{\sqrt{5}}{3}$

答案 A

6 若$x \in \left(-\frac{3\pi}{4},\frac{\pi}{4}\right)$，且$\cos\left(\frac{\pi}{4}-x\right) = -\frac{3}{5}$，则$\cos 2x$的值是（　）．

A. $-\frac{7}{25}$　　B. $-\frac{24}{25}$　　C. $\frac{24}{25}$　　D. $\frac{7}{25}$

答案　B

解析　$\cos 2x=\cos\left[\frac{\pi}{2}-2\left(\frac{\pi}{4}-x\right)\right]$

$=\sin 2\left(\frac{\pi}{4}-x\right)=2\sin\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-x\right)$,

$\because x\in\left(-\frac{3\pi}{4},\frac{\pi}{4}\right)$,

$\therefore \frac{\pi}{4}-x\in(0,\pi)$,

$\therefore \sin\left(\frac{\pi}{4}-x\right)0$,

又$\cos\left(\frac{\pi}{4}-x\right)=-\frac{3}{5}$,

$\therefore \sin\left(\frac{\pi}{4}-x\right)=\frac{4}{5}$,

$\therefore \cos 2x=2\sin\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-x\right)=-\frac{24}{25}$.

故选B .

7　已知$\cos\left(\alpha+\frac{\pi}{12}\right)=\frac{3}{5}$ ，则$\sin\left(2\alpha+\frac{2\pi}{3}\right)=$______ .

答案　$-\frac{7}{25}$

解析　$\because\cos\left(\alpha+\frac{\pi}{12}\right)=\frac{3}{5}$,

$\therefore\cos 2\left(\alpha+\frac{\pi}{12}\right)=2\cos^2\left(\alpha+\frac{\pi}{12}\right)-1=\frac{18}{25}-1=-\frac{7}{25}$,

即$\cos\left(2\alpha+\frac{\pi}{6}\right)=-\frac{7}{25}$,

$\therefore\sin\left(2\alpha+\frac{2}{3}\pi\right)=\sin\left(2\alpha+\frac{\pi}{6}+\frac{\pi}{2}\right)$

$=\cos\left(2\alpha+\frac{\pi}{6}\right)=-\frac{7}{25}$.

故答案为：$-\frac{7}{25}$.

2. 辅助角公式

8　化简$\frac{1}{\sin 15^\circ}-\frac{1}{\cos 15^\circ}$结果是（　）.

A. $\sqrt{2}$　　B. $\frac{\sqrt{2}}{2}$

C. $2\sqrt{2}$　　　　D. $-2\sqrt{2}$

答案 C

解析 由$\frac{1}{\sin 15^\circ}-\frac{1}{\cos 15^\circ}=\frac{\cos 15^\circ-\sin 15^\circ}{\sin 15^\circ\cos 15^\circ}=\frac{\sqrt{2}\cos(15^\circ+45^\circ)}{\frac{1}{2}\sin 30^\circ}=\frac{\frac{\sqrt{2}}{2}}{\frac{1}{4}}=2\sqrt{2}$.

故选C．

9 计算：$\sin\frac{7\pi}{12}\cos\frac{\pi}{6}+\cos\frac{5\pi}{12}\sin\frac{7\pi}{6}=$______．

答案 $\frac{\sqrt{2}}{2}$

解析 由诱导公式$\sin\frac{7\pi}{12}=\sin\left(\pi-\frac{5\pi}{12}\right)=\sin\frac{5\pi}{12}$，

$\sin\frac{7\pi}{6}=\sin\left(\pi+\frac{\pi}{6}\right)=-\sin\frac{\pi}{6}$，

得$\sin\frac{7\pi}{12}\cdot\cos\frac{\pi}{6}+\cos\frac{5\pi}{12}\cdot\sin\frac{7\pi}{6}$

$=\sin\frac{5}{12}\pi\cdot\cos\frac{\pi}{6}-\cos\frac{5\pi}{12}\sin\frac{\pi}{6}$

$=\sin\left(\frac{5\pi}{12}-\frac{\pi}{6}\right)$

$=\sin\frac{\pi}{4}$

$=\frac{\sqrt{2}}{2}$.

故答案为：$\frac{\sqrt{2}}{2}$．

10 已知$\sin\alpha+\cos\left(\alpha-\frac{\pi}{6}\right)=\frac{4\sqrt{3}}{5}$，则$\sin\left(\alpha+\frac{7\pi}{6}\right)$的值是______．

答案 $-\frac{4}{5}$

解析 $\sin\alpha+\cos\left(\alpha-\frac{\pi}{6}\right)$

$=\sin\alpha+\cos\alpha\cos\frac{\pi}{6}+\sin\alpha\cdot\sin\frac{\pi}{6}$

$=\frac{3}{2}\sin\alpha+\frac{\sqrt{3}}{2}\cos\alpha$

$=\sqrt{3}\left(\frac{\sqrt{3}}{2}\sin\ \alpha+\frac{1}{2}\cos\ \alpha\right)$

$=\sqrt{3}\left(\sin\ \alpha\cos\frac{\pi}{6}+\cos\ \alpha\sin\frac{\pi}{6}\right)$

$=\sqrt{3}\sin\left(\alpha+\frac{\pi}{6}\right)=\frac{4\sqrt{3}}{5}$.

$\therefore\sin\left(\alpha+\frac{\pi}{6}\right)=\frac{4}{5}$ ，

$\therefore\sin\left(\alpha+\frac{7\pi}{6}\right)=-\sin\left(\alpha+\frac{\pi}{6}\right)=-\frac{4}{5}$.

11 $\frac{\sin 10^\circ}{1-\sqrt{3}\tan 10^\circ}=$（　）．

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. $\frac{\sqrt{3}}{2}$

D. 1

答案 A

解析 $\frac{\sin 10^\circ}{1-\sqrt{3}\tan 10^\circ}=\frac{\sin 10^\circ\cos 10^\circ}{\cos 10^\circ-\sqrt{3}\sin 10^\circ}=\frac{2\sin 10^\circ\cos 10^\circ}{4\left(\frac{1}{2}\cos 10^\circ-\frac{\sqrt{3}}{2}\sin 10^\circ\right)}$

$=\frac{\sin 20^\circ}{4\sin(30^\circ-10^\circ)}=\frac{1}{4}$.

故选A．

3. 综合练习

12 已知$\alpha\in(0,\pi)$，且$3\cos 2\alpha-8\cos\alpha=5$，则$\sin\alpha=$（　）．

A. $\frac{\sqrt{5}}{3}$

B. $\frac{2}{3}$

C. $\frac{1}{3}$

D. $\frac{\sqrt{5}}{9}$

答案 A

解析 由$3\cos 2\alpha-8\cos\alpha=6\cos^2\alpha-8\cos\alpha-3=5$，

得$3\cos^2\alpha-4\cos\alpha-4=0$，

解得$\cos\alpha=-\frac{2}{3}$或$\cos\alpha=2$（舍），

又$\because\alpha\in(0,\pi)$，则$\sin\alpha>0$，

$\therefore \sin\alpha=\sqrt{1-\cos^2\alpha}=\dfrac{\sqrt{5}}{3}$.

故选A .

13 设θ为第二象限角，若$\tan(\theta+\dfrac{\pi}{4})=\dfrac{1}{2}$，则$\sin\theta+\cos\theta=$______ .

答案 $-\dfrac{\sqrt{10}}{5}$

14 已知α，β为锐角，且$\cos\alpha=\dfrac{7\sqrt{2}}{10}$，$\cos(\alpha+\beta)=\dfrac{2\sqrt{5}}{5}$，则$\cos 2\beta=$（　）.

A. $\dfrac{3}{5}$

B. $\dfrac{2}{3}$

C. $\dfrac{4}{5}$

D. $\dfrac{7\sqrt{2}}{10}$

答案 C

解析 $\sin^2\alpha=1-\cos^2\alpha=\dfrac{1}{50}$，又$\alpha$为锐角，$\sin\alpha>0$，

故$\sin\alpha=\dfrac{\sqrt{2}}{10}$，$\sin^2(\alpha+\beta)=1-\cos^2(\alpha+\beta)=\dfrac{1}{5}$，

又α，β为锐角，$\alpha+\beta\in(0,\pi)$，

$\therefore\sin(\alpha+\beta)>0$，$\sin(\alpha+\beta)=\dfrac{\sqrt{5}}{5}$，

$\cos\beta=\cos[(\alpha+\beta)-\alpha]=\dfrac{2\sqrt{5}}{5}\times\dfrac{7\sqrt{2}}{10}+\dfrac{\sqrt{2}}{10}\times\dfrac{\sqrt{5}}{5}=\dfrac{3\sqrt{10}}{10}$，

$\cos 2\beta=2\cos^2\beta-1=2\times\left(\dfrac{3\sqrt{10}}{10}\right)^2-1=\dfrac{4}{5}$.

故选C .

15 设α为锐角，若$\cos\left(\alpha+\dfrac{\pi}{6}\right)=\dfrac{4}{5}$，则$\sin\left(2\alpha+\dfrac{\pi}{3}\right)$的值为（　）.

A. $\dfrac{12}{25}$

B. $\dfrac{24}{25}$

C. $-\dfrac{24}{25}$

D. $-\dfrac{12}{25}$

答案 B

解析 本题主要考查三角函数 .

因为$\cos\left(\alpha+\dfrac{\pi}{6}\right)=\dfrac{4}{5}$，根据二倍角余弦公式，有$\cos\left(2\alpha+\dfrac{\pi}{3}\right)=2\cos^2\left(\alpha+\dfrac{\pi}{6}\right)-1=\dfrac{7}{25}$，

所以$\sin\left(2\alpha+\frac{\pi}{3}\right)=\pm\sqrt{1-\cos^2\left(2\alpha+\frac{\pi}{3}\right)}=\pm\frac{24}{25}$，

又因为α是锐角，所以$2\alpha+\frac{\pi}{3}\in\left(\frac{\pi}{3},\frac{\pi}{4}\right)$，

所以$\sin\left(2\alpha+\frac{\pi}{3}\right)\in\left(-\frac{\sqrt{3}}{2},1\right)$，所以$\sin\left(2\alpha+\frac{\pi}{3}\right)=\frac{24}{25}$．

故答案为$\mathbf{B}$．

16 已知$\theta\in(0,\pi)$，$\sin 2\theta=-\frac{24}{25}$，则$\sin\theta-\cos\theta=$（　）．

A. $\frac{7}{5}$

B. $-\frac{7}{5}$

C. $\pm\frac{7}{5}$

D. $\frac{1}{5}$

答案 A

解析 $\because\theta\in(0,\pi)$，且$\sin 2\theta=2\sin\theta\cdot\cos\theta=-\frac{24}{25}$，

$\therefore\sin\theta>0$，$\cos\theta<0$，

$\therefore\sin\theta-\cos\theta=\sqrt{(\sin\theta-\cos\theta)^2}=\sqrt{\sin^2\theta-2\sin\theta\cos\theta+\cos^2\theta}$

$=\sqrt{1+\frac{24}{25}}=\frac{7}{5}$．

故选$\mathbf{A}$．